

Sec. 6, is given by

$$a' = a_0 + bx, \quad (\text{A8})$$

where

$$x = \frac{1}{2}[(\cos^2\theta/\sin\theta) + \cos^2\theta/\theta].$$

Since the standard error of  $d$  and, therefore, of  $a'$  depends on the value of  $\theta$ , a weight inversely proportional to the variance of  $d$  was assigned to the  $a'$  values.

This weight was defined as

$$w_i = [1/(\sigma_{d_i}^*)^2]/\sum (1/\sigma_{d_i}^*)^2, \quad (\text{A9})$$

where

$$(\sigma_{d_i}^*)^2 = \sum_j [(d_{ij} - \bar{d}_i)^2/(n-1)] \quad (\text{A10})$$

and  $j = 1 \cdots n_1$ , the number of  $d$ -spacing determinations for each Bragg angle  $\theta$  and  $i = 1 \cdots k$ , the number of Bragg angles (see Fig. 6).

The values of  $a_0$  and  $b$  are obtained by minimizing the sum of the squares of the deviations:

$$S = \sum_i n_i w_i (a'_i - a_0 - bx_i)^2.$$

Again, using the results from weighted least squares, we have

$$b = \sum n_i w_i a'_i (x_i - \bar{x}) / \sum n_i w_i (x_i - \bar{x})^2 \quad (\text{A11})$$

and

$$a_0 = \bar{a}' - b\bar{x}, \quad (\text{A12})$$

where

$$\bar{a}' = \sum n_i a'_i w_i / \sum n_i w_i, \quad (\text{A13})$$

$$\bar{x} = \sum n_i x_i w_i / \sum n_i w_i. \quad (\text{A14})$$

The variance of estimate is given by

$$V(a'/x) = \sum n_i w_i (a'_i - a_0 - bx_i)^2 / (n-2), \quad (\text{A15})$$

where  $b$  and  $a_0$  are given by (A11) and (A12), respectively.

The variance of  $b$  and  $a_0$  are given by

$$V(b) = V(a'/x) / \sum n_i w_i (x_i - \bar{x})^2 \quad (\text{A16})$$

$$V(a_0) = V(a'/x) [(1/n) + \bar{x}^2 / \sum n_i w_i (x_i - \bar{x})^2]. \quad (\text{A17})$$

The standard errors are then

$$\sigma_b = [V(b)]^{1/2}, \quad (\text{A18})$$

$$\sigma_{a_0} = [V(a_0)]^{1/2}. \quad (\text{A19})$$

## APPENDIX B

### Computation of $d$ Spacings from Incomplete Ellipses

Referring to Fig. 1 it will be seen that

$$\begin{aligned} NP &= (a+b) \cot(\theta+\beta) + b \cot(\theta-\beta) \\ &= a \cot(\theta+\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)] \end{aligned} \quad (\text{B1})$$

$$\begin{aligned} NR &= b \cot(\theta+\beta) + (a+b) \cot(\theta-\beta) \\ &= a \cot(\theta-\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)]. \end{aligned} \quad (\text{B2})$$

Let  $NR = l$ , for brevity, and suppose that two patterns are superimposed on the same film by making two exposures and changing the distance  $a$  from the film to the target between the exposures. Then, for example, Eq. (B2) gives

$$l_1 = a_1 \cot(\theta-\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)], \quad (\text{B3})$$

$$l_2 = a_2 \cot(\theta-\beta) + b[\cot(\theta+\beta) + \cot(\theta-\beta)], \quad (\text{B4})$$

where the subscripts refer to the film position.

Subtracting (B4) from (B3),

$$\begin{aligned} l_1 - l_2 &= (a_1 - a_2) \cot(\theta-\beta) = c \cot(\theta-\beta) \\ \cot(\theta-\beta) &= (l_1 - l_2)/c = \Delta/c. \end{aligned} \quad (\text{B5})$$

Here  $c$  is the distance through which the film has been shifted between exposures and  $l_1 - l_2$  is the distance between corresponding ellipses measured on the film along their common major axis.

Let us now assume that the incident radiation consists of two components of wavelength  $\lambda_1$  and  $\lambda_2$ , respectively, each giving rise to a pattern. We have then from (B5)

$$\theta_1 - \beta = \text{arccot}(\Delta_1/c), \quad (\text{B6})$$

$$\theta_2 - \beta = \text{arccot}(\Delta_2/c), \quad (\text{B7})$$

where the subscripts now refer to different wavelengths. Subtracting (B7) from (B6),

$$\begin{aligned} \theta_1 - \theta_2 &= \text{arccot}(\Delta_1/c) - \text{arccot}(\Delta_2/c) \\ &= \text{arccot}[(c^2 + \Delta_1\Delta_2)/c(\Delta_1 - \Delta_2)] = \mu. \end{aligned} \quad (\text{B8})$$

Let

$$\lambda_1/\lambda_2 = \sin\theta_1/\sin\theta_2 = K,$$

then

$$\sin\theta_1 = \sin(\mu + \theta_2) = K \sin\theta_2. \quad (\text{B9})$$

The second of Eq. (B9) gives finally,

$$\cot\theta_2 = (K - \cos\mu)/\sin\mu, \quad (\text{B10})$$

where  $\mu$  is given by (B8).

The last expression gives the value of the Bragg angle (and therefore of the interplanar spacing) of a given set of planes in terms of the known x-ray wavelengths, of the film shift between exposures  $c$ , and of two quantities  $\Delta_1$  and  $\Delta_2$  measured on the film (Figs. 1 and 5).

Equation (B10) may be written in a form which gives directly the  $d$  spacings, obviates the need for trigonometric tables, and is convenient for computation on a desk calculator. Using the identity  $\sin\theta = (1 + \cot^2\theta)^{-1/2}$  and the value of  $\cot\theta_2$  given by (B10),

$$\begin{aligned} d &= \lambda_2/2 \sin\theta_2 = (\lambda_2/2) \\ &= (\lambda_2/2) \\ &= ((\lambda_2/2) \end{aligned}$$

Using the trigonometric identity  $\cot\mu = s$ , member of (B11)

$$1 + [(K/\sin\mu) - c]$$

A study of x-ray diffraction from thin films obtained by logographic exposure occurs in the cryogenic region. The investigation of the energy levels of the quasicrystalline materials is a subject of great interest in the field of condensed matter physics.

**STUDYING** microscopy and Weissmann's annealing plate-coherent with the (101) planes. Upon exposure or short an accumulated coherency of the relieved through parallel to the (101) planes. However, not possible, how the techniques quantify the distribution and to the observed

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